Closing Tue: Taylor Notes 1, 2, 3
Closing Thu: Taylor Notes 4, 5
Final is Saturday, March 12
5:00-7:50pm, KANE 130
Eight pages of questions, covers everything.

Recall:
The n^{th} Taylor Polynomial for f(x) based at x=b is given by:

$$T_n(x) = \sum_{k=0}^{n} \frac{1}{k!} f^{(k)}(b)(x-b)^k$$

and if $|f^{(n+1)}(x)| \leq M$, then

$$|f(x) - T_n(x)| \leq \frac{M}{(n+1)!}|x-b|^{n+1}$$

Entry Task:
Find the 9^{th} Taylor polynomial for $f(x) = e^x$ based at b = 0,
and give an error bound on the interval [-2,2]

TN 4: Taylor Series

Def'n: The **Taylor Series** for f(x) based at b is defined by

$$\sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(b)(x-b)^k = \lim_{n\to\infty} T_n(x)$$

If the limit exists a a particular value of *x*, then we say the series **converges** at *x*. Otherwise, we say it **diverges** at x.

The **open interval of convergence** gives the largest open interval of values at which the series converges.

Note: if

$$\lim_{n\to\infty} \frac{M}{(n+1)!}|x-b|^{n+1} = 0$$

then *x* is in the open interval of convergence.

A few patterns we know:

$$e^x = 1 + x + \frac{1}{2!}x^2 + \cdots = \sum_{k=0}^{\infty} \frac{1}{k!}x^k$$

$$\sin(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}x^{2k+1}$$

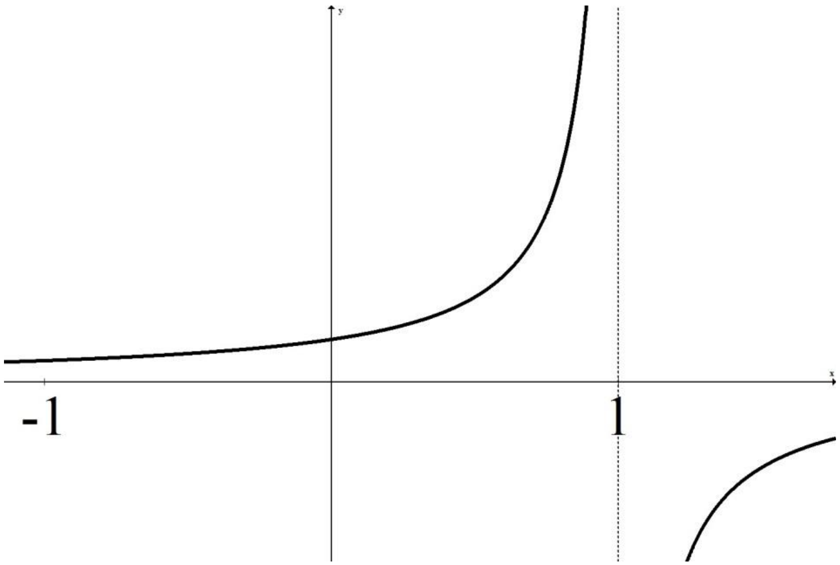
$$\cos(x) = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}x^{2k}$$

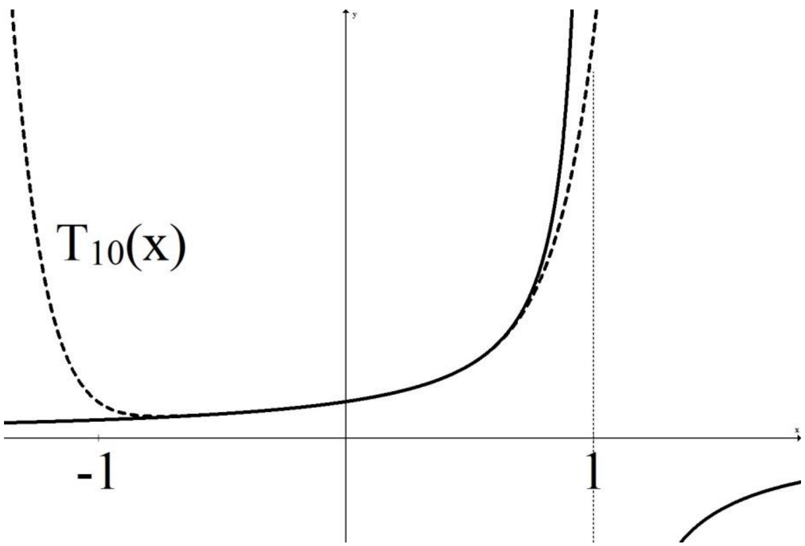
These converge for ALL values of *x*. So the **open interval of convergence** for each series above is (-∞,∞)

Now consider $f(x) = \frac{1}{1-x}$ based at x = 0.
Find the 10^{th} Taylor polynomial.
What is the error bound on [-1/2,1/2]?
What is the error bound on [-2,2]?

Graph of $y = 1/(1-x)$:



Graph of $y = 1/(1-x)$ and $T_{10}(x)$:



We will find all the following, and for these they converge for **-1 < x < 1**.

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots = \sum_{k=0}^{\infty} x^k$$

$$-\ln(1-x) = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots$$
$$= \sum_{k=0}^{\infty} \frac{1}{k+1} x^{k+1}$$

$$\arctan(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 + \cdots$$
$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1}$$

In other words, the open interval of convergence for these series is: -1 < x < 1.